Study of screening in QED_2 with new observables

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INNOVATIVE ECONOMY
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Outline

1. QED_2 in lightfront formulation
2. String tension
3. String breaking - screening
4. Summary

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - gA\!\!\!/ - m\right)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

[Eller, Pauli, Brodsky '87]:
In quantized model there are fermions $b_n^\dagger$ and antifermions $d_n^\dagger$.

$$Q = \sum_n (b_n^\dagger b_n - d_n^\dagger d_n) \quad \text{charge operator}$$

$$P^+ = \frac{2\pi}{L}\sum_n n(b_n^\dagger b_n + d_n^\dagger d_n) \equiv \frac{2\pi}{L}K \quad \text{momentum operator}$$

$$M^2 = m^2 K H_0 + \frac{g^2}{\pi} K V \quad \text{invariant mass}$$

- $K,\ Q,\ M^2$ commute $\rightarrow$ choose single K and Q and diagonalize M^2.
- M^2 is infinite for $Q \neq 0 \rightarrow$ take only states with $Q = 0$.
- for single K the mass operator is a finite matrix

Parametrization

$$M^2 = m^2 K H_0 + \frac{g^2}{\pi} K V = \tilde{m}^2 \left((1 - \lambda^2) K H_0 + \lambda^2 K V \right)$$

$\tilde{m}$ only scales $M^2 \to$ set $\tilde{m} = 1$

$$\lambda^2 = (1 + \pi (m/g)^2)^{-1}$$

$\lambda^2 \approx \frac{g^2}{\pi m^2}$ for small $\frac{g}{m}$ - weak coupling limit

$\lambda^2 \approx 1 - \frac{\pi m^2}{g^2}$ for small $\frac{m}{g}$ - small mass limit

Length scaling

$$P^+ = \frac{2\pi}{L} K$$

P^+	total momentum
K	cutoff
L	size of the system ($x \in (-L, L)$ with periodic bc)

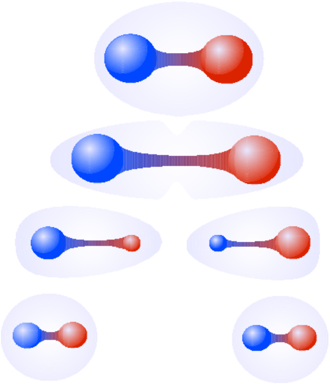
For constant P^+ length L scales with cutoff ($L \sim K$).

States in Hilbert space with cutoff K

$$|\mathbb{n}; \bar{\mathbb{n}}\rangle = |n_1, \ldots, n_N; \bar{n}_1, \ldots, \bar{n}_N\rangle = b^\dagger_{n_1} \ldots b^\dagger_{n_N} d^\dagger_{\bar{n}_1} \ldots d^\dagger_{\bar{n}_N} |\emptyset\rangle$$

$\sum n_i + \sum \bar{n}_i = K$

String picture



at small distances energy is proportional to the separation of particles $E = \sigma\Delta$

σ - string tension

at large distances it is favorable to create additional pair - string breaks

Idea:

- can study string tension in (dynamical) bound states in sector with two particles
- multiparticle states $\rightarrow$ screening $\rightarrow$ string breaking

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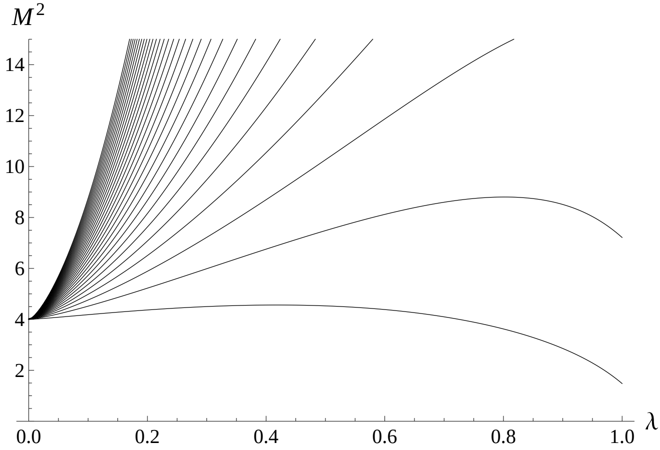
Masses in two particle sector



30 lowest mass states for different values of λ and $K = 400$.

Exclusive wavefunction in two particle sector (DFT)

$$|\phi\rangle = \sum_{n,\bar{n}} \tilde{\phi}_2(n,\bar{n})\,|n;\bar{n}\rangle + \sum_{\mathbb{n},\bar{\mathbb{n}}} \tilde{\phi}_4(\mathbb{n},\bar{\mathbb{n}})\,|\mathbb{n};\bar{\mathbb{n}}\rangle + \ldots$$

$$\mathbb{n} = (n_1, n_2)$$
$$n_1 < n_2$$

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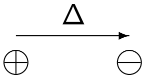
$$\mathbb{n} = (n_1, n_2) \qquad n_1 < n_2$$

$$\phi_2(\Delta) = \sum_n e^{-i(\xi_1 n + \xi_2 \bar{n})}\tilde{\phi}_2(n,\bar{n}) = e^{-iK\xi_2}\sum_n e^{i\Delta n}\tilde{\phi}_2(n,\bar{n})$$

$$\xi_i \in (-\pi, \pi)$$

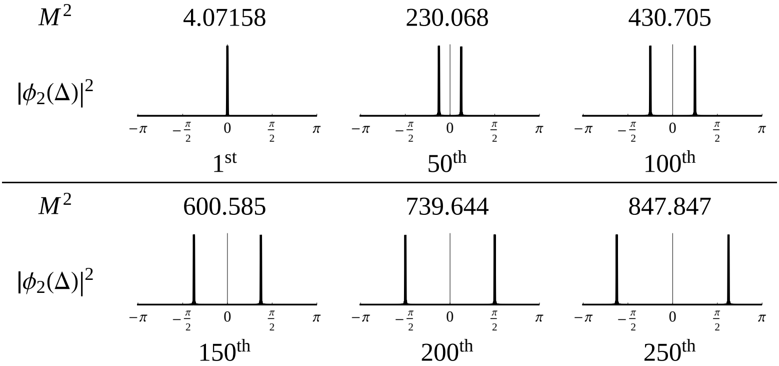


$|\phi_2(\Delta)|^2$ is probability distribution

of distance between fermion and antifermion

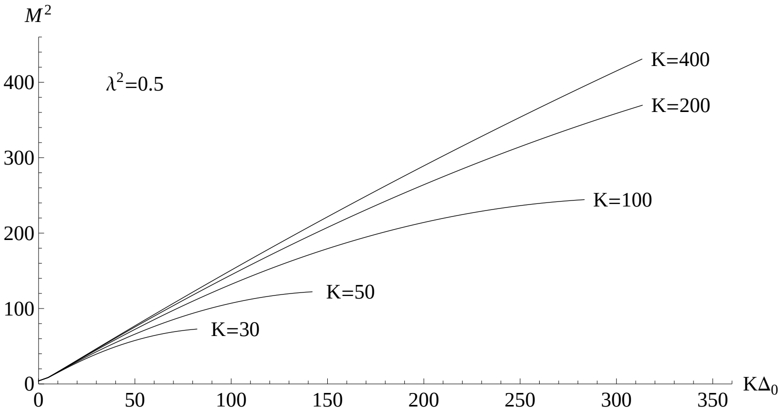
Two particle sector - masses and wavefunctions

parameters: $\lambda^2 = 0.5$, $K = 400$



Fermion is located at $\Delta = 0$. Antifermion is at $\Delta = \pm\Delta_0$.
Represent M^2 as a function of Δ_0.

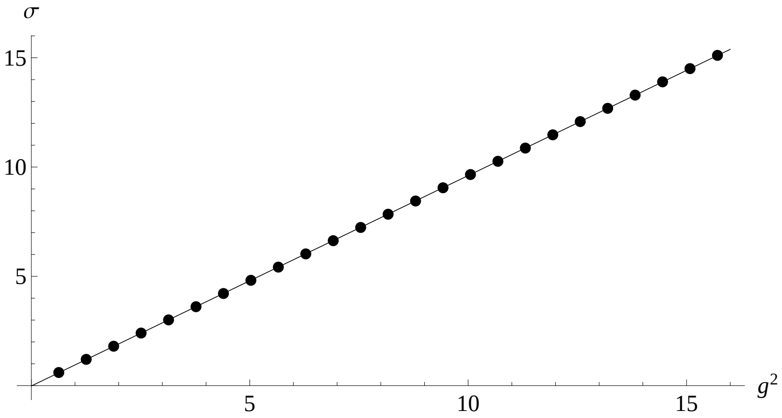
Mass dependence on pair separation



$M^2(\Delta_0)$ converges to a straight line for growing K

$M^2(\Delta_0) = m_0^2 + \sigma K \Delta_0 \Rightarrow \sigma \approx 1.5$

String tension as a function of g



Set $m = 1$. String tension is proportional to g^2.

$$\sigma \approx -0.01 + 0.96 g^2$$

Conclusions

Two particle sector:

- distance between fermion and antifermion is well defined
- M^2 is proportional to distance
- string tension: proportionality constant
- string tension is proportional to coupling constant

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How is this changed by multiparticle sectors?
Can this string be broken?

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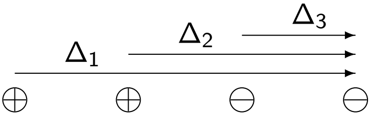
Inclusive functions

$$|\phi\rangle = \sum_{n,\bar{n}} \tilde{\phi}_2(n,\bar{n})\,|n;\bar{n}\rangle + \sum_{\mathbb{n},\bar{\mathbb{n}}} \tilde{\phi}_4(\mathbb{n},\bar{\mathbb{n}})\,|\mathbb{n};\bar{\mathbb{n}}\rangle + \dots$$

$$\mathbb{n} = (n_1, n_2)$$
$$n_1 < n_2$$

$$\phi_4(\Delta_1,\Delta_2,\Delta_3) = e^{-iK\xi_4} \sum_{n_1,n_2,\bar{n}_1} e^{i(\Delta_1 n_1 + \Delta_2 n_2 + \Delta_3 \bar{n}_1)}\,\tilde{\phi}_4(\mathbb{n},\bar{\mathbb{n}})$$



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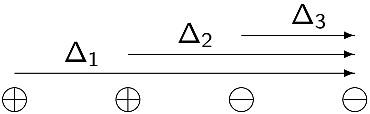


Function $|\phi_4(\Delta_1,\Delta_2,\Delta_3)|^2$ depends on 3 variables

$\rightarrow$ construct inclusive functions of one variable.

Inclusive functions

General formula: [Dorigoni, Veneziano, Wosiek '10]

$$D(\Delta) = \int d^{p-1}\vec{\Delta} \sum \delta(\Delta - \Delta_{ij}) |\phi_p(\vec{\Delta})|^2$$

Examples:

$D_{ff}(\Delta)$ probability distribution of a distance Δ **between two fermions**

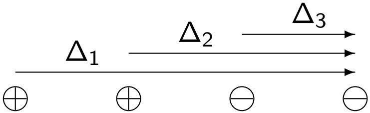
$D_{f\bar{f}}(\Delta)$ probability distribution of a distance Δ between one fermion and **any antifermion**

$D_{f\bar{f}}^{nn}(\Delta)$ probability distribution of a distance Δ between one fermion and **the nearest antifermion**

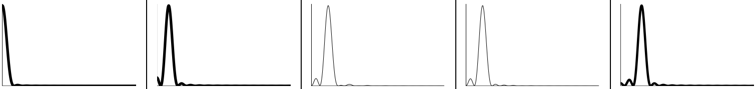
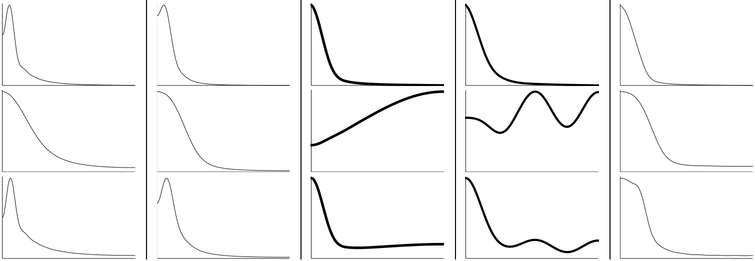
Inclusive functions

$$D_{ff}(\Delta) = \int d\Delta_1 d\Delta_2 d\Delta_3 \delta(\Delta - (\Delta_1 - \Delta_2)) |\phi_4(\Delta_1, \Delta_2, \Delta_3)|^2$$

$$D_{f\bar{f}}(\Delta) = \int d\Delta_1 d\Delta_2 d\Delta_3 \Big[\delta(\Delta - (\Delta_1 - \Delta_3)) + \delta(\Delta - \Delta_1) + \delta(\Delta - (\Delta_2 - \Delta_3)) + \delta(\Delta - \Delta_2)\Big] |\phi_4(\Delta_1, \Delta_2, \Delta_3)|^2$$



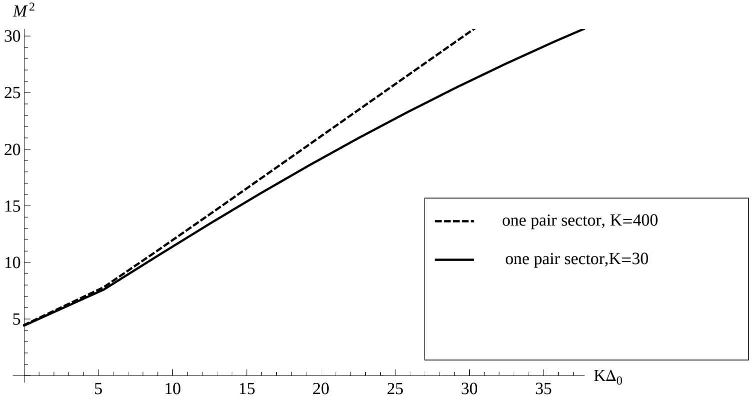
Inclusive functions ($K = 30,\ \lambda^2 = 0.5$)

		1st	3rd	4th	6th	12th
	M^2	3.97991	12.5317	15.5647	15.8564	20.3096
one pair	contribution	0.999928	0.98663	0.0270878	0.18004	0.86131
one pair	$\lvert\phi(\Delta)\rvert^2$					
two pairs	contribution	0.0000716186	0.0133694	0.972846	0.819909	0.138639
two pairs	$D^{nn}_{f\bar{f}}(\Delta)$, $D_{ff}(\Delta)$, $D_{f\bar{f}}(\Delta)$					

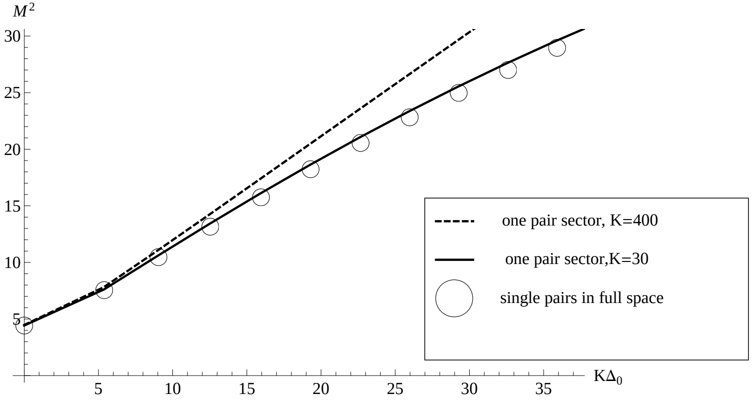
String remains at small distances.

At larger distances Δ_0 additional pair is created $\rightarrow$ screening $\rightarrow$ breaking of a string.

String breaking ($K = 30,\ \lambda^2 = 0.3$)



String breaking ($K = 30, \ \lambda^2 = 0.3$)



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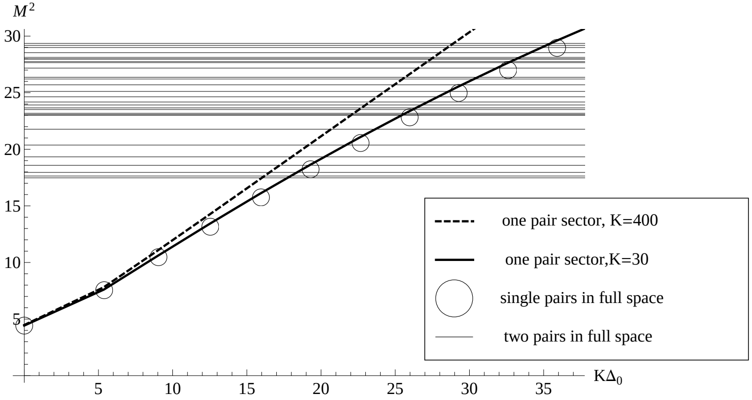

- at distance large enough the string is broken
- maximal length of the string shortens for growing λ

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Summary

- inclusive functions give insight into structure of the mass states
- at small distances M^2 is proportional to separation of two particles
- at some distance additional pair screens the interaction (state with 4 particles) $\Rightarrow$ string is broken
- additional pair screens the interaction
- string breaks earlier for stronger coupling / lower bare mass

Literature

- T. Eller, H.C. Pauli, S.J. Brodsky, *Discretized light cone quantization: The massless and the massive Schwinger model*, Phys.Rev.D 35(1987) 1493-1507
- D. Dorigoni, G. Veneziano, J. Wosiek, *Dimensionally reduced SYM_4 at large-N: an intriguing Coulomb approximation.*, JHEP 1106 (2011) 051

Inclusive function for the nearest neighbor

$$D^{nn}_{f\bar{f}}(\Delta) = \int d\Delta_1 d\Delta_2 \Bigg[\delta(\Delta - (\Delta_1 - \Delta_3)) \int_0^{2\pi - 2\Delta} d\Delta_3 + \delta(\Delta - \Delta_1) \int_{2\Delta}^{2\pi} d\Delta_3 + \delta(\Delta - (\Delta_2 - \Delta_3)) \int_0^{2\pi - 2\Delta} d\Delta_3 + \delta(\Delta - \Delta_2) \int_{2\Delta}^{2\pi} d\Delta_3 \Bigg] |\phi_4(\Delta_1, \Delta_2, \Delta_3)|^2$$